An iterative solution approach for truck routing and scheduling in the forest industry

Maximiliano Bordón¹, Jorge Marcelo Montagna¹, Gabriela Corsano¹
¹ Instituto de Desarrollo y Diseño, INGAR. UTN-CONICET.
Avellaneda 3657, Santa Fe, Argentina.
{mbordon, mmontagna, gcorsano}@santafe-conicet.gov.ar

Abstract. Log transportation in forest industry is a resource-intensive operation and represents a great challenge for logistic planners. Several trips must be generated in order to satisfy plants demand; in addition, trucks arrivals at each plant must be considered in order to avoid unproductive waiting times. In the Argentinean context, specifically in the Argentinian Northeast (NEA) region, these activities represent the main sustenance of the regional economies, so enhancing efficiency in the transport operation would represent a considerable improvement for these economies. In this work, an iterative solution approach for the truck routing and scheduling problems is presented. The proposed strategy involves two stages which are iteratively solved: product allocation, trip composition and truck routing problems are first solved through a Mixed-Integer Linear Programming model (MILP), while in the second stage, fixing the route for each truck according to the results of the previous step, a MILP model for the scheduling of truck arrivals at plants is considered. If no feasible solution for the scheduling problem is obtained, then an integer cut is applied in order to exclude from the search space truck routes already explored in previous iterations. The solution approach is tested in a case study representative of the Argentine context and conclusions are detailed.

Keywords: Vehicle Routing, Vehicle Scheduling, Forest, MILP.

1 Introduction

In forest industry, log transportation represents an important proportion in the cost structure of the companies, and therefore, its efficient management is a key issue. Transportation planning ranges from strategic to operational decisions: at the strategic level, decisions are mainly related to road investments, facilities locations and fleet management ([1], [2]); at the tactical level, product allocation from harvest areas to plants is carried out ([3], [4]); at the operational level, vehicle routing and scheduling are typical decisions.

Transportation at operational level in forest industry has the characteristic that a set of harvest areas supplies raw materials to few plants using a significant number of trucks. In general, several trucks make the same route, so it is common queues
generation at the plants and, therefore, unproductive waiting times. Generally, different types of raw materials are available in the harvest areas, with amounts that far exceed the quantities demanded by plants. Thus, in addition to routing and scheduling decisions, product assignment decisions must also be considered.

Log transportation problem is classified as a variation of the Pick-up and Delivery Problem (PDP). In [5] and [6] detailed definitions and approaches of general PDPs are presented. More information about the planning methods regarding the vehicle routing problem in the forest industry and optimization models can be found in [7] and [8]. When trucks arrivals must be synchronized with log loaders at plants, PDP is called Log Truck Scheduling Problem (LTSP). According with [9], LTSP consist of finding a set of minimum cost routes, one for each vehicle, in order to satisfy all customer demands and without exceeding the available supply. The authors associate the term “scheduling” with precedence constraints, where each truck has to pick-up material before any delivery: depending on vehicle characteristics (with or without crane), trucks arrivals scheduling (synchronization) at each node is not usually performed in the LTSP.

This kind of problems is classified as NP-hard, and different approaches are used to deal with it; most of them are heuristic based. A hierarchical approach to solve the weekly LTSP is proposed in [10]. In the first phase a MILP model is used to determine the destinations of full truckloads from forest areas to wood mills. In the second phase, two different methods to route and schedule the daily transportation of logs are used: a Constraint-Based Local Search approach (CBL), and a hybrid approach involving a Constraint Programming (CP) based model and a CBL model. Another hierarchical solution approach is presented in [11], where in the first stage a Linear Programming model (LP) for trips generation is used, while in a later stage a Tabu Search (TS) is developed for truck routing. In a similar fashion, unified TS for truck routing with pre-defined trips is applied in [12]. In [13] a weekly problem is solved through a three-stage approach: in the first phase, a large set of feasible routes is generated by CP; in the second phase, the assignment of routes to trucks is made by a MILP model; finally, in the last phase, the scheduling of trucks is solved by CP. The drawback of these previous works is that due to route representation, the truck assignment problem results very complex to be solved in reasonable computing time, and therefore heuristic approaches were proposed, which cannot assure the global optimal solution.

Unlike the previously mentioned works, in this paper an iterative solution approach to solve log truck routing and scheduling problem through an exact approach is presented. In this work, an extension of the model developed by [14] is introduced. The proposed approach consists of an iterative procedure that involves two phases: in the first one, trips generation and truck routing decisions are jointly considered using a MILP model; and in the second one, scheduling of truck arrivals is solved through a MILP model. In the first stage, the objective is to minimize the total transportation cost, while in the second one the objective is to minimize the excess over the maximum working time of each truck, i.e., maximum route duration. If any excess is reached, a cut is applied and a new iteration is made. The procedure finishes when no truck exceeds its working time achieving an optimal solution, or when a predefined number of iterations is reached, giving the best found feasible solution.
In the following section, the problem to be modeled is described. In section 3, the proposed approach is presented, while in section 4, the case study and the obtained results are shown. Finally, in section 5 conclusions are exposed.

2 Problem statement

The problem considered in this work involves plants $i$, $i \in I$, which requires different types of raw material $m$, $m \in M$, taking into account the manufactured products. $DEM_{i,m}$ represents the amount of raw material type $m$ demanded by plant $i$, measured in full-truckloads. Raw materials are provided by a set of harvest areas $f, f \in F$, each one with a maximum available capacity of each type of raw material $(OF_{f,m})$, in full-truckloads.

A set of regional bases $p, p \in P$, from which departs and returns trucks $c, c \in C_p$, is available. Each truck $c$ can transport a single type of raw material per trip (simultaneous hauling of different types of raw material is not allowed) and, due to transport regulations, they can make a limited number of trips on the day. Besides, each truck begins and ends its route in the same regional base.

If a truck is used, then a fixed fee must be paid. For each used truck a time limit is fixed, i.e., maximum working hours, but no limit about the waiting time at plants for unloading materials is determined. In the case that the truck exceeds its stipulated working hours, this additional time must be paid, measured in $ per exceeded hour.

Each route is composed by a series of trips $v, v \in V$, where a trip $v$ is a sequence of different movements (Fig. 1). Many times the number of trips included in a route is limited by certain regulations. For example, a route can include at most three trips or cycles, despite the truck can have time to make more trips. Four types of trips are defined:

a) If the route is composed by only one trip $v$, it has a departure movement (unloaded) from the regional base $p$ to a harvest area $f$, a loaded movement from the harvest area $f$ to plant $i$, and a return movement (unloaded) from that plant $i$ to the regional base $p$.

b) If the route has more than one trip, its first trip is composed by a departure movement (unloaded) from the regional base $p$ to a harvest area $f$, a loaded movement from that harvest area $f$ to plant $i$, and an unloaded movement from that plant $i$ to a harvest area $f'$ (not necessarily equal to $f$) to make a new loaded trip.

c) If the trip is neither the last one nor the first one of the route, then it is composed by a loaded movement from harvest area $f$ to a plant $i$ and an unloaded movement from that plant $i$ to a harvest area $f'$ (not necessarily equal to $f$) to make a new loaded trip.

d) If the trip is the last one for the truck route but is not the first one, then it is composed by a loaded movement from harvest area $f$ to plant $i$ and a return movement (unloaded) from that plant $i$ to the regional base $p$. 
In Fig. 1 the involved variables are also presented: $DT_{c,p,f,v}$ takes value 1 if truck $c$ departs from regional base $p$ to harvest area $f$ in the trip $v$. $LT_{c,f,i,v}$ represents the realization of the loaded movement made by truck $c$ from harvest area $f$ to plant $i$ during trip $v$. $UT_{c,f',i,v}$ describes if the unloaded movement made by truck $c$ from $i$ to $f'$ is completed during trip $v$, and $RT_{c,p,i,v}$ takes value 1 if the movement from plant $i$ to regional base $p$ is made by truck $c$ in trip $v$. As can be seen, the first two types of trips are made up by three movements, two unloaded and another loaded, meanwhile the two remaining are conformed by two movements, one unloaded and another loaded. Therefore, according to the above trip definition, the proposed approach builds the routes through trip compositions which are simultaneously assigned to the route in the routing model. Taking into account the four types previously defined, all possible route can be made up using them. Fig. 2 shows two routes composed by two and three trips, respectively.

Given the information of average speeds of trucks (which depend on road infrastructure and load status of truck), the distances between pairs of nodes, raw materials availability on harvest areas, plants demand, opening and closing times of plants, cost per traveled kilometers, fixed costs per truck used, maximum working time of trucks, release time of trucks and trucks availability, it is desired to determine:

a) Product allocation, i.e., what harvest area supplies what plant with which raw material.

b) Assignment of trips to trucks.

c) Routing decisions, i.e., sequence of trips in each truck.

d) Scheduling of trucks arrivals at each plant.

The aim is to minimize the overall transportation cost, composed by cost per traveled kilometers and cost per truck used, subject to plants requirements and maximum available time of trucks.
3 Solution approach

As was mentioned before, the proposed approach consists of an iterative procedure that includes two phases: in the first one, routing decisions are taken in order to minimize transportation costs, while in the second, truck arrivals are scheduled in order to minimize the excess on the maximum route duration. In particular, routing model consists of the minimization of Eq. (1), i.e. the total transportation cost. Some constraints are associated with: supply and demand satisfaction equations, Eq. (2) and (3), respectively; maximum route duration equations (Eq. (4) and (5)); and route composition equations (Eq. (6) to (9)).

\[
\text{minimize } TCOST
\]

Where:

\[
TCOST = \sum_{c \in C} \sum_{f \in F} \sum_{i \in I} \sum_{v \in V} CL_{f,i}DF_{f,i}LT_{c,f,i,v} + \\
\sum_{c \in C} \sum_{f \in F} \sum_{i \in I} \sum_{v \in V} CU_{a_{f,i}}DF_{f,i}UT_{c,f,i,v} + \\
\sum_{p \in P} \sum_{c \in C} \sum_{f \in F} \sum_{i \in I} \sum_{v \in V} CU_{b_{p,f}}DPF_{p,f}DT_{c,p,f,v} + \\
\sum_{p \in P} \sum_{c \in C} \sum_{i \in I} \sum_{v \in V} CU_{c_{p,i}}DPI_{p,i}RT_{c,p,i,v} + \sum_{c \in C} C_{\text{truck}}_{c} Y_{c}
\]
Parameters $DFI_{f,i}$, $DPF_{p,f}$, and $DPI_{p,i}$ represent the distances between the harvest area $f$ and plant $i$, between the regional base $p$ and harvest area $f$, and between the regional base $p$ and plant $i$, respectively. $CL_{f,i}$ represents the cost of traveling with load across the respective path, while $CU_{a_{f,i}}$, $CU_{b_{p,f}}$, and $CU_{c_{p,i}}$ define the costs of traveling without load through the involved nodes, all of these parameters expressed in $ per kilometer. $C_{truck}$ states the fixed cost of using truck $c$ and $Y_c$ is the binary variable that takes the value 1 if truck $c$ is used.

Equations (2) and (3) state that the number of loaded movements (in full-truckloads) must not exceed the available raw material at harvest areas and fulfill the plants demands, respectively:

\[
\sum_{c \in C} \sum_{v \in V} \sum_{i \in I_m} LT_{c,f,i,v} \leq OF_{f,m}, \forall f \in F, \forall m \in M \\
\sum_{c \in C} \sum_{v \in V} \sum_{f \in F} LT_{c,f,i,v} = DEM_{i,m} \forall m \in M, \forall i \in I_m
\]  

Equations (2) and (3) state that the number of loaded movements (in full-truckloads) must not exceed the available raw material at harvest areas and fulfill the plants demands, respectively:

\[
\sum_{c \in C} \sum_{v \in V} \sum_{i \in I_m} LT_{c,f,i,v} \leq OF_{f,m}, \forall f \in F, \forall m \in M \\
\sum_{c \in C} \sum_{v \in V} \sum_{f \in F} LT_{c,f,i,v} = DEM_{i,m} \forall m \in M, \forall i \in I_m
\]  

Here, $I_m$ represents the set of plants $i$ that require raw material $m$.

Equation (4) defines the total working time of each truck, $TIME_c$, which depends on road network and average speeds with and without load. These times must not exceed the available working hours of each truck, $MaxT_c$, (Eq. (5)):

\[
TIME_c = \sum_{v \in V} \sum_{f \in F} \frac{DPF_{p,f}}{VUT_{a_{p,f}} - DT_{c,p,f,v}} + \\
\sum_{v \in V} \sum_{f \in F} \sum_{i \in I} \frac{DFI_{f,i}}{VLT_{f,i}} LT_{c,f,i,v} + \frac{DFI_{f,i}}{VUT_{f,i}} UT_{c,f,i,v} + \\
\sum_{v \in V} \sum_{f \in F} \sum_{i \in I} \frac{DPI_{p,i}}{VUT_{C_{p,i}}} RT_{c,p,i,v} + \\
\sum_{v \in V} \sum_{f \in F} \sum_{i \in I} \left(Unload_{c,i} + Load_{c,f}\right) LT_{c,f,i,v} \forall p \in P, \forall c \in C_p
\]

\[
TIME_c \leq MaxT_c \quad \forall c \in C
\]  

$VLT_{f,i}$ represents the average speed with load, while $VUT_{a_{p,f}}$, $VUT_{b_{f,i}}$, and $VUT_{C_{p,i}}$ define the average speeds without load. $Load_{c,f}$ and $Unload_{c,i}$ are the needed time to load a truck at harvest area $f$ and to unload truck at plant $i$, respectively.

If truck $c$ is used then it must depart and return to the regional base where it began (Eq. (6) and (7), respectively) and make at least one loaded movement (Eq. (8)). In addition, if truck $c$ completes a loaded movement then it can either make an unloaded movement to make a new loaded movement or return to the associated regional base (Eq. (9)):

\[
\sum_{f \in F} DT_{c,p,f,v} = Y_c \quad \forall p \in P, \forall c \in C_p, \forall v = 1
\]
In order to reduce the alternative solutions and improve the model performance, some constraints like ordered truck allocation and routes allocation in decreasing form with respect to traveled distance for trucks belonging to the same regional base, among others, are included in the model but for space reasons are not here displayed.

Once the routing model is solved, some variables and sets are fixed as input data in the scheduling model, i.e., variables $DT_{c,p,i,v}$, $LT_{c,f,i,v}$, $RT_{c,p,i,v}$, and $Y_{c}$. Moreover, the following sets are created: $C_{used}$ which represents the subset of trucks that were used in the routing model, and $V_{ci}$ which defines the subset of trips that were made by each truck $c$ arriving at plant $i$ and belonging to set $C_{used}$.

Regarding the scheduling model, a continuous time representation based on time slots is used. The aim is to minimize Eq. (10) subject to: assignment constraints of loaded movements to slots (Eq. (11) and (12)); starting and ending times of trip $v$ of truck $c$ (Eq. (13) to (15)); due date restrictions (Eq. (16)); starting time of trip $v$ of truck $c$ must coincide with starting time of slot $s$ of plant $i$ (Eq. (17) and (18)); earliest and latest starting times of trip $v$ of truck $c$ (Eq. (19)); total working hours of truck $c$ (Eq. (20)); excess on maximum route duration (Eq. (21)); and others constraints related to arrival times, starting and ending times of slots that, for space reasons, are not presented.

Equation (10) states that the model aim is to minimize the excess on route duration. The second term is added in order to obtain solutions with lower waiting times.

\[
\text{minimize } Z \quad (10)
\]

where:

\[
Z = \sum_{c \in C_{used}} C_{Excess} \cdot Excess_{c} + \sum_{c \in C_{used}} \sum_{v \in V_{ci}} C_{WT} \cdot WT_{c,v}
\]

The continuous variables $Excess_{c}$ represents the time that truck $c$ exceeds the fixed maximum route duration, and $WT_{c,v}$ is the time that truck $c$ has to wait for unload at plant during trip $v$ (waiting time). Parameters $C_{Excess}$ and $C_{WT}$ represent the cost per exceeded time and the cost per waiting time, respectively.

Equation (11) states that each trip $v$ for each selected truck $c$ must be allocated to one and only one slot $s$, while Eq. (12) establishes that slot $s$ of plant $i$ can be used only once.
\[
\sum_{c \in C_{\text{used}}} \sum_{v \in V_{ci}} Y_{s,c,i,v,s} = 1 \quad \forall i \in I, \quad \forall s \in S_i
\]  

(12)

where \( S_i \) is the set of slots \( s \) of plant \( i \), and its cardinality is equal to the amount of full-truckloads that demands each plant \( i \). In addition, \( Y_{s,c,i,v,s} \) is the binary variable that takes the value 1 if the trip \( v \) of truck \( c \) is made in slot \( s \) of plant \( i \).

As the routes are fixed in the scheduling model, the arrival time to the first plant \( i \) depends on the needed time to arrive to that plant \( (ARR_{ci}) \) and the variable \( ST_{c,i} \), which represents the staring time for truck \( c \). Let \( ST_{c,i} \) be the time in which the truck \( c \) starts the unloading operation during trip \( v \), which is equal to the arrival time at plant \( i \) \( (AT_{ci}) \) and the time that the truck must wait for unload \( (WT_{ci}) \). Then Eq. (13) defines \( ST_{c,i} \) for the first trip, while Eq. (14) for the following ones, since they depend on the loading time on harvest areas:

\[
ST_{c,i} = ARR_{c,v} + ST_{c} + WT_{c,v} \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}, \quad v = 1
\]  

(13)

\[
ST_{c,i} = AT_{c,v} + WT_{c,v} + \sum_{f \in F} \sum_{i \in I} \sum_{s \in S_i} \text{Load}_{c,f,LT_{c,f,i,v}} \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}, \quad v > 1
\]  

(14)

Equation (15) defines the ending time of trip \( v \) of truck \( c \) \( (ET_{c,v}) \) and Eq. (16) states that all the trips must finish before closing time of plant \( i \) \( (Close_i) \):

\[
ET_{c,v} = ST_{c,v} + \sum_{i \in I} \sum_{s \in S_i} \text{Unload}_{c,i} Y_{s,c,i,v,s} \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}
\]  

(15)

\[
ET_{c,v} \leq Close_i \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}
\]  

(16)

Equations (17) and (18) establish that the starting time of trip \( v \) of truck \( c \) must be equal to starting time of slot \( s \) of plant \( i \) \( (ST_{s,i}) \) through a Big-M formulation.

\[
ST_{c,v} - ST_{s,i} \geq B(M(Y_{s,c,i,v,s} - 1)) \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}, \forall i \in I, \forall s \in S_i
\]  

(17)

\[
ST_{c,v} - ST_{s,i} \leq B(M(1 - Y_{s,c,i,v,s})) \quad \forall c \in C_{\text{used}}, \forall v \in V_{ci}, \forall i \in I, \forall s \in S_i
\]  

(18)

Earliest \( (EST_c) \) and latest \( (LST_c) \) starting times of truck \( c \) are stated in Eq. (19), while the total using time of truck \( c \) \( (USE_c) \) is defined in Eq. (20).

\[
EST_c \leq ST_c \leq LST_c \quad \forall c \in C_{\text{used}}
\]  

(19)

\[
USE_c = ET_{c,v} - ST_c + \sum_{i \in I} \sum_{p \in P} \text{DIT}_{p,i} \text{RT}_{c,p,i,v} \quad \forall p \in P, \forall c \in C_{\text{used}} \cap C_{p,v} \in V_{last_c}
\]  

(20)

\( V_{last_c} \) is the set that includes the last trip \( v \) made by truck \( c \) belonging to \( C_{\text{used}} \).
The excess time with respect to maximum route duration is given by Eq. (21).

\[ \text{Excess}_c \geq \text{USE}_c - \text{MaxT}_c, \quad \forall c \in C_{\text{used}} \] (21)

Thus, the proposed approach consists in a two-phase iteration that includes the models mentioned above (Fig. 3).

Basically, the algorithm proceeds as follows: first, the model parameters are initialized and the iteration is set at value 1; then, the routing model is executed to obtain a feasible solution to the problem of products assignment to plants, trips generation, trips allocation to trucks, and trips sequencing. If this execution does not provide a feasible solution, the algorithm ends. The infeasibilities are related to the data structure (such as not enough trucks, not enough supply, or even distances so long that any route exceeds the maximum working time).

In case the routing model gives a feasible solution, the obtained routing variables are fixed and necessary sets are constructed. With this input data, the scheduling model is solved. In this stage, a solution with no excesses in the overall time routes is searched. If the optimal solution has exceeded time for any truck route, or even if all the arrivals cannot be programmed in the plant time window, a cut constraint is added, in order to avoid repeated solutions. These cuts restrictions are given by:

\[ \sum_{(c,f,i,v) \in \text{Cut}_{1}^{\text{iter}}} LT_{c,f,i,v} - \sum_{(c,f,i,v) \in \text{Cut}_{0}^{\text{iter}}} LT_{c,f,i,v} \leq |\text{Cut}_{1}^{\text{iter}}| - 1 \] (22)

where:

\[ \text{Cut}_{1}^{\text{iter}} = \{(c,f,i,v) : LT_{c,f,i,v} = 1 \text{ in the optimal solution of routing model at iteration } \text{iter}\} \]

\[ \text{Cut}_{0}^{\text{iter}} = \{(c,f,i,v) : LT_{c,f,i,v} = 0 \text{ in the optimal solution of routing model at iteration } \text{iter}\} \]

Constraints (22) states that if a truck \( c \) exceeds its maximum available time, in the next iteration at least one of the generated movements must be modified. For further details on (22), refer to [15].

Before adding integer cuts into the routing model, the obtained solution (with excess) is saved in the solution pool and iteration \( \text{iter} \) is increased by 1.

After integer cut constraint (Eq. (22)) is added to the routing model, this MILP model is again solved, repeating iteratively the previous steps. If the number of iterations of the algorithm reaches its maximum level, the procedure provides the best found solution (with excess) and ends.

In case the scheduling model provides a solution without excess in all routes, the procedure ends providing the optimal solution, i.e., minimize the maximum excess on routes duration.
To illustrate the application and performance of the proposed approach, a case study adapted from one of the majors companies of the region is presented.

The supply chain considered is composed by 2 plants (\(i_1\): sawmill, \(i_2\): pulp and paper mill) that demands 2 types of raw material (\(m_1\): sawable raw material, \(m_2\): pulpable raw material), 8 harvest areas and 3 regional bases. The firm operates with a truck homogeneous fleet composed by 40 trucks. Distances between nodes are stated in Table 1, while raw material availability at harvest areas and plants demand are shown in Table 2. Each truck can be in duty at most 8 hours and its earlier (EST) and
latest \( (LST_i) \) starting time are 6:45 h. and 8:15 h., respectively. Each plant can receive raw material between 8:00 and 20:00 h.

The algorithm was implemented in GAMS [16] 24.7.4 version, using CPLEX 12.6.3 solver in an Intel(R) Core(TM) i7-4790, 3.60 GHz, 8 GB RAM.

### Table 1. Distances between nodes (in kilometers).

<table>
<thead>
<tr>
<th></th>
<th>( i_1 )</th>
<th>( i_2 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( f_4 )</th>
<th>( f_5 )</th>
<th>( f_6 )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_1 )</td>
<td>-</td>
<td>-</td>
<td>45</td>
<td>29</td>
<td>50</td>
<td>70</td>
<td>95</td>
<td>38</td>
<td>60</td>
<td>97</td>
<td>44</td>
</tr>
<tr>
<td>( i_2 )</td>
<td>-</td>
<td>-</td>
<td>93</td>
<td>77</td>
<td>60</td>
<td>40</td>
<td>39</td>
<td>77</td>
<td>111</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>44</td>
<td>99</td>
<td>52</td>
<td>49</td>
<td>65</td>
<td>94</td>
<td>116</td>
<td>47</td>
<td>56</td>
<td>121</td>
<td>-</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>52</td>
<td>45</td>
<td>69</td>
<td>51</td>
<td>45</td>
<td>64</td>
<td>56</td>
<td>86</td>
<td>66</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>93</td>
<td>36</td>
<td>107</td>
<td>92</td>
<td>76</td>
<td>49</td>
<td>32</td>
<td>93</td>
<td>125</td>
<td>32</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 2. Raw material availability and plants demand (in full-truckloads).

<table>
<thead>
<tr>
<th></th>
<th>( i_1 )</th>
<th>( i_2 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( f_4 )</th>
<th>( f_5 )</th>
<th>( f_6 )</th>
<th>( f_7 )</th>
<th>( f_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>19</td>
<td>-</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>-</td>
<td>21</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

In Table 3 obtained results are presented. Only two iterations are needed to arrive at the optimal solution and 160.77 s. were necessary to complete the procedure. In the optimal solution 14 trucks are used, with a total cost of $90,112.55, that is, the sum of total routing costs ($90,020) and total scheduling (waiting) costs ($92.55).

### Table 3. Algorithm iterations and obtained results.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Execution time (accumulated) (s)</th>
<th>Total iteration excess (h)</th>
<th>Waiting time (h)</th>
<th>Total routing costs ($)</th>
<th>Fixed routing costs ($)</th>
<th>Variable routing costs ($)</th>
<th>Waiting costs ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iter₁</td>
<td>72.80</td>
<td>0.15</td>
<td>7.19</td>
<td>90,020</td>
<td>9,100</td>
<td>80,920</td>
<td>107.85</td>
</tr>
<tr>
<td>Iter₂</td>
<td>160.77</td>
<td>0.00</td>
<td>6.17</td>
<td>90,020</td>
<td>9,100</td>
<td>80,920</td>
<td>92.55</td>
</tr>
</tbody>
</table>

In Table 4 detailed iterations information are given. In order to simplify the reading of the table, an identifier for each generated route is used, namely, \( r_1, r_2, \ldots, r_n \).
In both iterations the same trucks are used, although the generated routes on the last iteration are different with respect to the first one. Comparing second iteration with the previous one, the routes that remain are \( r_3, r_7, r_8 \) and \( r_{11} \), but now they are performed by different trucks, while in the second iteration 7 new routes are generated, exchanging trips within the routes and between the routes.

In this case, only two iterations are needed for finding the optimal solution, but in general, the developed algorithm allows finding the optimal solution to the given problem in a sufficiently large number of iterations. A feasible solution is guaranteed as long as enough number of trucks is available.

The cut equation allows different route configurations. By requiring that in each iteration at least one loaded movement must be modified, different situations can occur:

(a) Exchanges of loaded movements within the same route, while the others routes remain without changes;

(b) A route incorporates loaded movements and the others routes exchanges loaded movements between them;

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Routes Generated</th>
<th>ID Route</th>
<th>Assigned Trucks</th>
<th>Route Excess (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iter_1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_1 )</td>
<td>( c_1 )</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_2 )</td>
<td>( c_2 )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_3 )</td>
<td>( c_3 )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_4 )</td>
<td>( c_{11} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_5 )</td>
<td>( c_{12}, c_{14}, c_{15} )</td>
<td>0.00/0.00/0.00</td>
</tr>
<tr>
<td>Iter_2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{12} )</td>
<td>( c_1 )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_3 )</td>
<td>( c_2 )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{13} )</td>
<td>( c_3 )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{14} )</td>
<td>( c_{11} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( r_4 )</td>
<td>( c_{12}, c_{13}, c_{14} )</td>
<td>0.00/0.00/0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{15} )</td>
<td>( c_{15} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_6 )</td>
<td>( c_{16} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_8 )</td>
<td>( c_{36} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{16} )</td>
<td>( c_{37} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{17} )</td>
<td>( c_{38} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( p_1, f_1, f_1, f_1, f_1, f_1, f_1, p_1 )</td>
<td>( r_{18} )</td>
<td>( c_{39} )</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( r_{11} )</td>
<td>( c_{40} )</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
(c) All generated routes are different with respect to the previous iteration;
(d) New trucks are used (or some trucks are no longer needed at subsequent iterations) and therefore, one or more routes change.

The only case that is not allowed by the cut equation is that all the generated routes are the same, since in the routing model there are equations that assign the longest routes in a decreasing manner to the trucks that belong to the same regional base.

In the analyzed case study, it can be seen that a truck (c₁) modifies its route (r₁) making a new one (r₁₂). In this particular case, r₁ is no longer used, but can be used by another truck. For example, c₂ makes the route r₃ that was made by c₁ in a previous iteration.

In addition, it can happen that a truck does not change at all the route that it made in a previous iteration, as is the case of c₃₆ that makes the route r₈ in both iterations. It can also happen that new routes are generated from the current ones through exchanges of some trips between them. For example, routes r₉ and r₁₀ generated in the first iteration and the routes r₁₆ and r₁₈ generated in the second iteration are similar but the first loaded movement of each one was exchanged between them.

5 Conclusions and future work

In forestry operations, transportation planning is a key issue. In this work, an iterative solution approach for truck routing and scheduling was presented. The potentiality of the proposed approach lies in the possibility of solving exactly real-size instances in a reasonable computational time.

Unlike the previous published works in the literature, the presented approach allows taking decisions such as trip generation, truck allocation, routing and scheduling decisions and solves the overall formulation in a short computational time. A case study representative of the Argentine context was analyzed to showing the capabilities of the solution approach and promises results were obtained.

The proposed approach makes use of the advantages provided both by the routing model and the scheduling model. With respect to the routing model, the benefits are associated with a reduction in the combinatorial degree of the problem, i.e., the proposed formulation generates routes as a succession of trips and, in turn, these trips as a concatenation of movements, which allows a significant reduction in the number of generated binary variables.

Regarding the scheduling model, time slot based representation allows to reduce the scheduling decisions to an allocation of trucks to plant slots.

In the presented example, one of the objectives of the cut equation can be visualized: not eliminate or prohibit alternative solutions. In fact, as can be seen, one of these alternative solutions turned out to be the optimal solution in a later iteration for the presented problem.
Since log transportation depends on the level harvested in the harvest areas, it would be useful to integrate routing and scheduling with bucking decisions, i.e., to solve harvesting and transportation planning in an integrated way. In order to do this, as a future task, the proposed approach will be extended to a multi-period formulation.

References